COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on ''Quantum processes in the field of a two-frequency circularly polarized plane electromagnetic wave''

N. B. Narozhny and M. S. Fofanov
Department of Theoretical Physics, Moscow State Engineering Physics Institute (Technical University), Kashirskoe Shosse 31, Moscow 115409, Russia

(Received 30 June 1998)

We show that there is a mistake in the results recently published by An Yu and H. Takahashi [Phys. Rev. E **57**, 2276 (1998)] for the probabilities of a photon emission by an electron and a pair production by a photon in the field of a two-frequency plane electromagnetic wave. In this paper we present the corrected expression for the probability of a photon emission which contains terms missed by Yu and Takahashi. We argue also that the effect of presence of the waves with combination frequencies in the external field proposed by the authors of the paper [Phys. Rev. E **57**, 2276 (1998)] has no physical basis. [S1063-651X(99)10008-4]

PACS number(s): 41.60.−m, 12.20.−m, 32.80.Wr, 23.20.Ra

I. INTRODUCTION

In their recent paper [1] An Yu and H. Takahashi presented a derivation of the probabilities of photon emission by electron and pair production by a photon in the field of a two-frequency plane electromagnetic wave. Unfortunately, there is a mistake in their calculations. Both the matrix element S_{fi} [Eq. (3.2) of [1]] and the total scattering rate W [Eq. (3.10) of [1]] are represented as sums over four Fourier indices s_1,s_2,s_3,s_4. This could be possible if while calculating the square of the matrix element $|S_{fi}|^2$ validity of the following formula was assumed:

$$\begin{aligned}&\delta^{(4)}[q+s_1k_1+s_2k_2+s_3(k_1-k_2)+s_4(k_1+k_2)-q'-k']\\ &\quad\times\delta^{(4)}[q+s_1'k_1+s_2'k_2+s_3'(k_1-k_2)\\ &\quad+s_4'(k_1+k_2)-q'-k']\\ &=\frac{VT}{(2\pi)^4}\delta^{(4)}[q+s_1k_1+s_2k_2+s_3(k_1-k_2)\\ &\quad+s_4(k_1+k_2)-q'-k']\delta_{s_1s_1'}\delta_{s_2s_2'}\delta_{s_3s_3'}\delta_{s_4s_4'}.\end{aligned}\tag{1}$$

VT in Eq. (1) means 4-volume of integration and the other notation have the same meaning as in Ref. [1].

It is true that the product of two δ functions on the left-hand side of Eq. (1) is not equal to zero only if their arguments coincide. But the equalities for indices $s_i=s_i'$, $i=1-4$ are sufficient but not necessary conditions for such coincidence. It is easy to see that the arguments of δ functions on the left-hand side of Eq. (1) coincide under the conditions

$$s_1+s_3+s_4=s_1'+s_3'+s_4',\quad s_2-s_3+s_4=s_2'-s_3'+s_4',$$

if η $(k_2=\eta k_1)$ is an irrational number and

$$s_1+s_3+s_4+\eta(s_2-s_3+s_4)=s_1'+s_3'+s_4'+\eta(s_2'-s_3'+s_4'),$$

if η is rational. Therefore, the right-hand side of Eq. (1) should be written as follows:

$$\frac{VT}{(2\pi)^4}\Lambda_{n_1n_2;n_1'n_2'}\delta^{(4)}[q+n_1k_1+n_2k_2-q'-k'],\tag{2}$$

where

$$\Lambda_{n_1n_2;n_1'n_2'}=\begin{cases}\delta_{n_1n_1'}\delta_{n_2n_2'}, & \eta\ \text{irrational},\\ \delta_{n_1+\eta n_2,n_1'+\eta n_2'}, & \eta\ \text{rational},\end{cases}\tag{3}$$

and

$$\begin{aligned}n_1&=s_1+s_3+s_4,\\ n_2&=s_2-s_3+s_4.\end{aligned}\tag{4}$$

It is clear now that the square of the matrix element and the total scattering rate are represented in the general case as sums over at least six different Fourier indices and hence an infinite number of ''nondiagonal'' terms are lost in Eq. (3.10) of Ref. [1].

The interpretation of the probabilities in terms of wave ''photons'' proposed by the authors of Ref. [1] cannot be considered satisfactory either. By analogy with the case of a plane monochromatic wave [2,3], the authors interpret indices s_1,s_2,s_3,s_4 in Eq. (3.2) of [1] as numbers of ''photons'' with respective frequencies ω_1, ω_2, $\omega_1-\omega_2$, $\omega_1+\omega_2$ absorbed from the wave $(s_i>0)$ or emitted to the wave $(s_i<0)$ by the electron. On this basis they discuss the possibil-

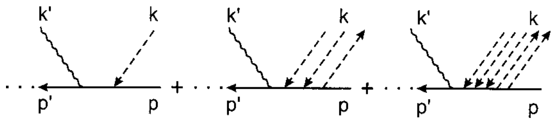

FIG. 1. Diagrams representing the partial amplitude M_1 of photon emission by an electron in the field of a monochromatic wave.

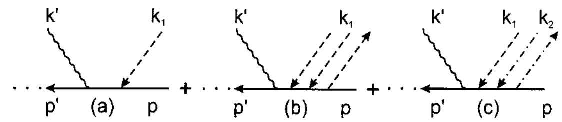

FIG. 3. Diagrams for photon emission with absorption of one photon from the first mode. Dashed lines correspond to the mode with k_1. Dotted and dashed lines correspond to the mode k_2.

ity of observation of a third color with one of the combination frequencies $\omega_1 \mp \omega_2$ in the initially two-frequency external field. However, no physical mechanism is offered to explain this effect. Our understanding is that there could be only two mechanisms for generating waves with combination frequencies. The first is nonlinear interaction of monochromatic components of the external field due to vacuum polarization. But this effect is beyond the framework of the author's consideration. Besides, it is well known that no vacuum polarization effects exist in the field of a plane electromagnetic wave of arbitrary intensity and spectral distribution [4]. The other is emission of such waves by the electron. But the electron according to QED principles is allowed to emit only final state photons with frequency ω'.

Lack of a mechanism of third color generation is not surprising. Introduction of the concept of photons needs something more than expandability of the matrix element in Fourier series. Moreover, it can often be done in different ways. For example the matrix element of a photon emission by an electron in a plane monochromatic wave can be represented as a double Fourier series but it does not mean at all that there exist photons of two types in the external field. Even in this case identification of the index s with the number of absorbed (emitted) photons is based not only on the opportunity for Fourier expansion of the S-matrix element but primarily on the fact that the partial amplitudes M_s can be reproduced in the frames of Feynman diagram technique; see Refs. [2,3] and especially [5]. The last statement means literally the following. M_s can be constructed in correspondence with the rules of standard Feynman technique where the factor $e_\mu e^{-ikx}$ ($e_\mu^* e^{ikx}$) is associated with every absorbed (emitted) photon belonging to the monochromatic wave with wave 4-vector k_μ. The polarization 4-vector e_μ is determined by the configuration of the external field. For a linearly polarized monochromatic wave with the 4-potential $A_\mu = a_\mu \cos kx$ $e_\mu = a_\mu/2$, for a circularly polarized wave with the 4-potential $A_\mu = a_{1\mu} \cos kx + a_{2\mu} \sin kx$ $e_\mu = (a_{1\mu} + i a_{2\mu})/2$, and so on.

In particular, the amplitude M_1 is represented by diagrams of Fig. 1, where dots mean all the permutations of vertices. Note that this technique allows one to reconstruct

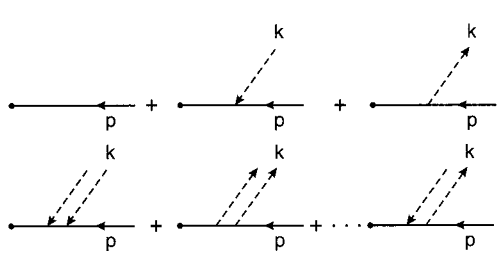

FIG. 2. Diagrams representing the Volkov solution in the field of a plane monochromatic wave.

also the Volkov solution; see Ref. [5]. The diagrams representing the solution $\Psi_p(x)$ are shown in Fig. 2.

We will show in this paper that in the case of a two-frequency wave the same correspondence between the partial amplitudes and diagram technique exists if numbers n_1 and n_2 introduced in Eq. (4) are considered as the number of photons belonging to monochromatic modes with wave 4-vectors k_1 and k_2, respectively. So there is no need to introduce any mysterious ''combination photons'' for interpretation of the results.

For the sake of compactness we will consider in this paper only photon emission by an electron in a circularly polarized two-frequency plane electromagnetic wave with 4-potential in the form coinciding with Eq. (3.12) of [1]:

$$\begin{gathered} A = A_1 + A_2, \\ A_1 = a_1 \cos\varphi_1 + a_2 \sin\varphi_1, \quad \varphi_1 = k_1 x, \\ A_2 = \zeta[a_1 \cos(\varphi_2 + \varphi) \pm a_2 \sin(\varphi_2 + \varphi)], \quad \varphi_2 = k_2 x, \\ \varphi = \text{const}, \quad k_1^2 = 0, \\ k_1 a_1 = k_2 a_2 = 0, \quad a_1^2 = a_2^2, \quad a_1 a_2 = 0, \\ k_2 = \eta k_1, \quad \eta < 1. \end{gathered} \tag{5}$$

This will be done in Sec. II. Section III describes correspondence between partial probabilities derived from exact expressions in the limit of weak field and those calculated in the framework of perturbation theory. Results obtained are discussed in Sec. IV.

We use relativistic units $\hbar = c = 1$ throughout the paper.

II. THE PROBABILITY OF PHOTON EMISSION BY AN ELECTRON

The S-matrix element for emission of a photon with momentum $k' = (\omega', \mathbf{k}')$ and polarization e' by an electron is equal to

$$S_{fi} = -ie \int \bar{\Psi}_{p'} (\gamma e'^*) \Psi_p \frac{\sqrt{4\pi} e^{ik'x}}{\sqrt{2\omega'}} d^4x, \tag{6}$$

where Ψ_p and $\Psi_{p'}$ are the wave functions of the electron in the initial and the final states, respectively.

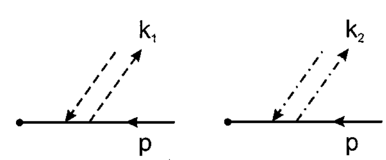

FIG. 4. Elements causing divergences in the diagram technique.

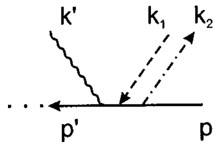

FIG. 5. Diagrams for photon emission with absorption of one photon from mode k_1 and emission of one photon to mode k_2.

For the field with the 4-potential (5) the wave function Ψ_p (Volkov solution) is easily obtained by substitution of the potential A from Eq. (5) into the expression for the solution of the Dirac equation in the field of an arbitrary plane electromagnetic wave given by Eq. (40.7) in Ref. [6]. It has the form

$$\Psi_p=\left[1+\frac{e(\gamma k_1)(\gamma A_1)}{2(pk_1)}+\frac{e(\gamma k_2)(\gamma A_2)}{2(pk_2)}\right]\frac{u(p)}{2q_0}\times\exp\{-i[qx+R_{1p}+R_{2p}+R_{3p}]\},$$

$$R_{1p}=\frac{e(a_1p)}{(pk_1)}\sin\varphi_1-\frac{e(a_2p)}{(pk_1)}\cos\varphi_1,$$

$$R_{2p}=\zeta\left(\frac{e(a_1p)}{(pk_2)}\sin(\varphi_2+\varphi)\mp\frac{e(a_2p)}{(pk_2)}\cos(\varphi_2+\varphi)\right),\tag{7}$$

$$R_{3p}=-\frac{e^2a_1^2}{(pk_1)}\frac{\zeta}{1\mp\eta}\sin[\varphi_1\mp(\varphi_2+\varphi)],$$

and coincides with Eq. (2.5) of [1] for the case $\varphi=0$, $a_3=\zeta a_1$, $a_4=\pm\zeta a_2$.

The integrand of the expression (6) is a combination of products of the following factors:

$$e^{i(R_{1p'}-R_{1p})}\{1,\cos\varphi_1,\sin\varphi_1\},$$

$$e^{i(R_{2p'}-R_{2p})}\{1,\cos(\varphi_2+\varphi),\sin(\varphi_2+\varphi)\},$$

$$e^{i(R_{3p'}-R_{3p})}\{1,\cos(\varphi_1\mp(\varphi_2+\varphi))\}.$$

Expanding each of them in Fourier series following the formula (101.7) in Ref. [6] we can represent S_{fi} as a threefold sum

$$S_{fi}=\frac{1}{\sqrt{2q_0 2q_0' 2\omega'}}\sum_{s_1,s_2,s_3}M_{fi}^{(s_1,s_2,s_3)}(2\pi)^4\times\delta^{(4)}[q+s_1k_1+s_2k_2-s_3(k_1\mp k_2)-q'-k'].\tag{8}$$

Note that if we introduce numbers

$$n_1=s_1-s_3,\qquad n_2=s_2\pm s_3,\tag{9}$$

we can rewrite Eq. (8) in the form

$$S_{fi}=\frac{1}{\sqrt{2q_0 2q_0' 2\omega'}}\sum_{n_1,n_2}(2\pi)^4\times\delta^{(4)}[q+n_1k_1+n_2k_2-q'-k']\tilde{M}_{fi}(n_1,n_2),\tag{10}$$

where

$$\tilde{M}_{fi}(n_1,n_2)=\sum_{s_3=-\infty}^{\infty}M_{fi}^{(n_1+s_3,n_2\mp s_3,s_3)}.\tag{11}$$

It seems reasonable now to interpret n_1 and n_2 in Eq. (10) as the numbers of photons absorbed by the electron from the modes of the external field with the wave 4-vectors k_1 and k_2, respectively. And $\tilde{M}_{fi}(n_1,n_2)$ can be interpreted then as a partial amplitude of photon emission owing to absorption by the electron n_1 photons from the first and n_2 photons from the second mode. We will show later that this interpretation is not contradictory and agrees with interpretation based on the Feynman diagram technique.

For the total probability of photon emission summed over polarizations of the particles in finite state and averaged over polarizations of the initial electron we find

$$W^{\pm}=\frac{1}{2\pi}\sum_{s_1,s_2,s_3}\sum_{s_1',s_2',s_3'}\Lambda_{n_1n_2;n_1'n_2'}\int\frac{d^3k'}{\omega'}\frac{d^3q'}{q_0'}\delta^{(4)}[q+n_1k_1+n_2k_2-q'-k']\times e^{i[n_1-n_1'\pm(n_2-n_2')]\varphi_0+i(n_2-n_2')\varphi}w^{\pm}(s_1,s_2,s_3;s_1',s_2',s_3'),\tag{12}$$

where the symbol $\Lambda_{n_1n_2;n_1'n_2'}$ is defined in Eq. (3),

$$\cos\varphi_0=\frac{\alpha_1}{\sqrt{\alpha_1^2+\alpha_2^2}},\quad \sin\varphi_0=\frac{\alpha_2}{\sqrt{\alpha_1^2+\alpha_2^2}},\quad \alpha_i=\frac{e(a_ip)}{(k_1p)}-\frac{e(a_ip')}{(k_1p')},\tag{13}$$

and $w^{\pm}(s_1,s_2,s_3;s_1',s_2',s_3')$ has the form

$$w^{\pm}(s_1,s_2,s_3;s_1',s_2',s_3')=\frac{e^2m^2}{q_0}\left[\prod_{i=1}^{3}J_{s_i}(z_i)J_{s_i'}(z_i)\right]\left\{-1+\xi_1^2\left(1+\frac{u^2}{2(1+u)}\right)\left[-(1+\zeta^2)-\zeta\frac{s_3+s_3'}{z_3}\right.\right.$$
$$+\frac{1}{2}\left(\frac{J_{s_1+1}(z_1)}{J_{s_1}(z_1)}+\zeta\frac{J_{s_2\pm1}(z_2)}{J_{s_2}(z_2)}\right)\left(\frac{J_{s_1'+1}(z_1)}{J_{s_1'}(z_1)}+\zeta\frac{J_{s_2'\pm1}(z_2)}{J_{s_2'}(z_2)}\right)+\frac{1}{2}\left(\frac{J_{s_1-1}(z_1)}{J_{s_1}(z_1)}+\zeta\frac{J_{s_2\mp1}(z_2)}{J_{s_2}(z_2)}\right)$$
$$\left.\left.\times\left(\frac{J_{s_1'-1}(z_1)}{J_{s_1'}(z_1)}+\zeta\frac{J_{s_2'\mp1}(z_2)}{J_{s_2'}(z_2)}\right)\right]\right\}. \tag{14}$$

We used in Eq. (14) the following notation:

$$u=\frac{(k_1k')}{(k_1p')},\quad \xi_1^2=-\frac{e^2a_1^2}{m^2},\quad \xi^2=\xi_1^2+\xi_2^2,\quad \xi_2=\zeta\xi_1,\quad m_*^2=m^2(1+\xi^2),$$
$$z_1=\sqrt{\alpha_1^2+\alpha_2^2}=2v\frac{\xi_1}{\sqrt{1+\xi^2}}\sqrt{\frac{u}{u_v}\left(1-\frac{u}{u_v}\right)},\quad z_2=\frac{\zeta}{\eta}z_1,$$
$$z_3=2v\frac{\xi_1^2\zeta}{(1\mp\eta)(1+\xi^2)}\frac{u}{u_v},\quad u_v=2v\frac{(k_1p)}{m_*^2},\quad v=n_1+\eta n_2. \tag{15}$$

It is convenient to perform integrations in Eq. (12) choosing the center-of-mass frame for every fixed value of parameter $v=n_1+\eta n_2$ in which

$$\mathbf{q}+v\mathbf{k_1}=\mathbf{q}'+\mathbf{k}'=\mathbf{0}. \tag{16}$$

We choose the coordinate system and the gauge in that frame so that the vectors $\mathbf{a_1}$ and $\mathbf{a_2}$ are along axes 1 and 2 respectively, $\mathbf{k_1}$ is along axis, 3 and $a_{10}=a_{20}=0$. Then after integration over $\mathbf{q}'$ with the help of δ functions we have

$$\int\frac{d^3k'}{\omega'}\frac{d^3q'}{q_0'}\delta^{(4)}[q+vk_1-q'-k']\{\cdots\}$$
$$=2\int\frac{dk'^1dk'^2dk'_-}{k'_-(q_--k'_-)}\delta\left[\frac{k_\perp'^2q_-}{k'_-(q_--k'_-)}+\frac{m_*^2}{q_--k'_-}-q_+-2v\omega\right]\{\cdots\}, \tag{17}$$

where we introduced "±" components of a 4-vector y^μ as $y_\pm=y^0\pm y^3$, and $\mathbf{k}_\perp'^2=(k'^1)^2+(k'^2)^2$. After the change of the variables on the right-hand side of Eq. (17),

$$k'^1=k'_\perp\cos\psi,\quad k'^2=k'_\perp\sin\psi,\quad k'_-=q_-\frac{u}{1+u},$$

and integration over $k'_\perp$ with the help of the δ function, we get

$$\int\frac{d^3k'}{\omega'}\frac{d^3q'}{q_0'}\delta^{(4)}[q+vk_1-q'-k']\{\cdots\}$$
$$=\int_0^{u_v}\frac{du}{(1+u)^2}\int_0^{2\pi}d\psi\{\cdots\}. \tag{18}$$

It is easy to show that in the chosen reference frame

$$\cos\psi=-\cos\varphi_0,\quad \sin\psi=-\sin\varphi_0,$$

and hence $\psi=\varphi_0+\pi$. Therefore, finally we have

$$\int\frac{d^3k'}{\omega'}\frac{d^3q'}{q_0'}\delta^{(4)}[q+vk_1-q'-k']\{\cdots\}$$
$$=\int_0^{u_v}\frac{du}{(1+u)^2}\int_0^{2\pi}d\varphi_0\{\cdots\}. \tag{19}$$

The integrand in Eq. (12) depends on φ_0 only through the phase factor. Therefore, integration over φ_0 can be easily carried out and yields the factor

$$2\pi\delta_{n_1\pm n_2,n_1'\pm n_2'}.$$

Since

$$\Lambda_{n_1n_2;n_1'n_2'}\delta_{n_1\pm n_2,n_1'\pm n_2'}=\delta_{n_1n_1'}\delta_{n_2n_2'}, \tag{20}$$

independently of whether η is rational or irrational, we finally find for the total probability of photon emission

$$W^{\pm}=\sum_{n_1,n_2=-\infty}^{\infty}\theta(n_1+\eta n_2)W^{\pm}(n_1,n_2), \tag{21}$$

$$W^{\pm}(n_1,n_2)=\sum_{s_3,s_3'=-\infty}^{\infty}\int_0^{u_v}\frac{du}{(1+u)^2}\times w^{\pm}(n_1+s_3,n_2\mp s_3,s_3;n_1+s_3',n_2\mp s_3',s_3'). \quad (22)$$

The presence of the Heaviside θ function

$$\theta(x)=\begin{cases}1, & x>0\\ 0, & x\leqslant 0\end{cases}$$

in Eq. (21) is a consequence of the kinematic equation

$$q+vk_1=q'+k',$$

which allows the electron to absorb only positive momentum from the external field in units of k_1,

$$v=n_1+\eta n_2=\frac{(q'k')}{(qk_1)}>0. \quad (23)$$

Our expression for the total probability of photon emission given by Eqs. (21), (22), (14) aside from diagonal terms with $s_3=s_3'$ which exactly coincide with the probability found in Ref. [1] [Eqs. (5.1) and (3.16)] contains an infinite number of nondiagonal terms missed by the authors of Ref. [1].

III. THE LIMIT OF WEAK FIELD AND PERTURBATION THEORY

In the preceding section we have offered an interpretation of n_1 and n_2 in Eqs. (10) and (21) as the numbers of photons absorbed from the first or the second mode of the external field. If this interpretation is correct, the quantities $W^{\pm}(n_1,n_2)$ given by Eq. (22) have the meaning of the partial probabilities of photon emission with absorption of n_1 photons from the first and n_2 photons from the second mode. Thus $W^{\pm}(1,0)$ should be considered as the probability of photon emission with absorption of only one photon from the first mode. Diagrams corresponding to this process are shown in Fig. 3. In the limit of weak field we find from Eqs. (14) and (22)

$$W^{\pm}(1,0)=\frac{e^2m^2\xi_1^2}{4q_0}\int_0^{u_1}\frac{du}{(1+u)^2}\left\{2+\frac{u^2}{1+u}-4\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)-4\xi_1^2\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\left[1+\frac{u^2}{1+u}-\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\right]\right\}$$
$$+\frac{e^2m^2}{2q_0}\frac{\xi_1^4\zeta^2}{\eta^2}\int_0^{u_1}\frac{du}{(1+u)^2}\frac{u}{u_1}\left[-\frac{\eta^2}{1\mp\eta}\left(2+\frac{u^2}{1+u}\right)+\left(1-\frac{u}{u_1}\right)\left(-2(1\pm 2\eta)-(1\pm\eta)^2\frac{u^2}{1+u}\right.\right.$$
$$\left.\left.+4\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\pm\frac{4\eta}{1\mp\eta}\frac{u}{u_1}\right)\right]. \quad (24)$$

The first integral in Eq. (24) repeats the result obtained in Ref. [3] for the case of the monochromatic wave (the term of order ξ_1^2 is the Klein-Nishina formula). The only difference is that u_1 now depends on the effective mass $m_*=m\sqrt{1+\xi_1^2+\xi_2^2}$ which feels the presence of the second mode.

It could seem that the first integral in Eq. (24) is determined by diagrams (a) and (b) in Fig. 3 just as in the monochromatic field and therefore its dependence on ξ_2 cannot be explained in the framework of the diagram technique. Nevertheless, it is not so. Among diagrams of the types (b) and (c) of Fig. 3 there are divergent diagrams containing elements shown in Fig. 4. Nikishov and Ritus proposed a renormalization procedure which removes these divergences [5]. According to this procedure diagrams shown in Fig. 4 reduce to free electron lines with simultaneous replacement of a free electron momentum p by a quasimomentum q, or bare mass m by effective mass m_*. Just because of this effect, the diagrams of type (b) in Fig. 3 contribute to the first integral in Eq. (24). It is clear also that this contribution changes only u_1 but not the structure of the probability itself.

The second integral in Eq. (24) is the result of interference of diagrams (a) and (c) in Fig. 3.

It is of interest that only the diagonal terms $s_3=s_3'=0$ in the sum in Eq. (22) contribute to the first integral in Eq. (24). Hence it can be derived from the expression for the total probability presented by An Yu and Takahashi. The interference term cannot be reproduced from their result.

The term of order ξ_1^4 in Eq. (21) involves many parts besides those we have discussed already. We give here the formula for the probability $W^{\pm}(1,-1)$ which cannot be calculated without nondiagonal terms $s_3\neq s_3'$ in the sum in Eq. (22):

$$W^{\pm}(1,-1)=\frac{e^2m^2}{4q_0}\xi_1^4\zeta^2\int_0^{u_v}\frac{du}{(1+u)^2}\left\{(1\pm 1)\frac{u}{u_v}\left[2-2\frac{u}{u_v}+\frac{u^2}{1+u}\right]+\frac{(1-\eta)^2}{\eta^2}\frac{u}{u_v}\left(1-\frac{u}{u_v}\right)\right.$$
$$\left.\times\left[(1+\eta^2)\left(2+\frac{u^2}{1+u}\right)-4(1-\eta)^2\frac{u}{u_v}\left(1-\frac{u}{u_v}\right)+\eta(1\mp 1)\left(2+\frac{u^2}{1+u}\right)-(1\pm 1)\eta\left(4\frac{u}{u_v}-2-\frac{u^2}{1+u}\right)\right]\right\}, \quad (25)$$

where $u_v = 2(1-\eta)(k_1 p)/m_*^2$. The diagrams shown in Fig. 5 describe this probability.

We will calculate now the probability $W^{\pm}(1,-1)$ in the perturbation theory. The amplitude corresponding to six diagrams shown in Fig. 5 according to the rules of Feynman diagram technique has the form

$$iM_{fi} = (-ie)^3 \sqrt{4\pi}\,\overline{u}(p')e'^* O u(p), \tag{26}$$

$$\begin{aligned} e'^* O = &(\gamma e'^*)G(p+k_1(1-\eta))(\gamma e_1)G(p-\eta k_1)(\gamma e_2^*) + (\gamma e_1)G(p'-k_1)(\gamma e'^*)G(p-\eta k_1)(\gamma e_2^*) \\ &+ (\gamma e_1)G(p'-k_1)(\gamma e_2^*)G(p'-(1-\eta)k_1)(\gamma e'^*) + (\gamma e'^*)G(p+k_1(1-\eta))(\gamma e_2^*)G(p+k_1)(\gamma e_1) \\ &+ (\gamma e_2^*)G(p'+\eta k_1)(\gamma e'^*)G(p+k_1)(\gamma e_1) + (\gamma e_2^*)G(p'+\eta k_1)(\gamma e_1)G(p'-(1-\eta)k_1)(\gamma e'^*), \end{aligned} \tag{27}$$

where $G(q)$ is an electron propagator, and polarization 4-vectors are defined as

$$e_1 = \frac{a_1 + i a_2}{2}, \quad e_2 = \zeta \frac{a_1 \mp i a_2}{2}. \tag{28}$$

Taking into account that $(\gamma p)u(p) = m u(p)$, $\overline{u}(p')(\gamma p') = m\overline{u}(p')$ the expression for O can be simplified and reduced to

$$\begin{aligned} e^2 O^{\mu} = &\left[-\frac{z_1 z_2}{4} e^{i(1\mp 1)\varphi_0} - \frac{1\pm 1}{2}\zeta \xi_1^2 \frac{u}{u_v} \right]\gamma^{\mu} + \frac{1\pm 1}{4}\zeta\xi_1^2 \frac{m^2}{(pk_1)^2}(1+u)(\gamma k_1)k_1^{\mu} + \frac{z_1}{4}e^{i\varphi_0}\left[\frac{e\gamma^{\mu}(\gamma k_1)(\gamma e_2^*)}{(k_1 p)} + \frac{e(\gamma e_2^*)(\gamma k_1)\gamma^{\mu}}{(k_1 p')} \right] \\ &- \frac{z_2}{4} e^{\mp i\varphi_0}\left[\frac{e\gamma^{\mu}(\gamma k_1)(\gamma e_1)}{(k_1 p)} + \frac{e(\gamma e_1)(\gamma k_1)\gamma^{\mu}}{(k_1 p')} \right], \end{aligned} \tag{29}$$

with $z_1, z_2, \varphi_0, u, u_v$ the same as in Eq. (15) and $v = 1-\eta$.

The probability of photon emission summed over polarizations of the particles in the final state and averaged over polarizations of the initial electron $W^{\pm}(1,-1)$ is equal to

$$\begin{aligned} W^{\pm}(1,-1) = &-\frac{e^2}{16\pi p_0}\int \frac{d^3k'}{\omega'}\frac{d^3p'}{p_0'}\delta[p+(1-\eta)k_1 - p' \\ &- k']e^4 \,\mathrm{Tr}\{(m+(\gamma p))\overline{O}^{\mu}(m+(\gamma p'))O_{\mu}\}. \end{aligned} \tag{30}$$

After calculating the trace and performing integrations in Eq. (30) by the same method described in the preceding section, we get exactly the formula given by Eq. (25). This result confirms our interpretation of the process in the field of a two-frequency plane wave in terms of wave photons.

IV. DISCUSSION

In this paper we have derived the probability of photon emission by an electron in the field of a two-frequency plane electromagnetic wave. This result corrects a mistake made by the authors of Ref. [1]. The total probability is represented as a double sum of partial probabilities $W^{\pm}(n_1, n_2)$ of photon emission with absorption (or emission) n_1 photons from one and n_2 photons from another mode, Eq. (21). The partial probabilities themselves are represented as double infinite sums over Fourier indices s_3, s_3', Eq. (22).

We did not give here the expression for the probability of pair production but it can be easily derived from Eq. (12) after substitutions $p \to -p$, $k' \to -k_{\gamma}$, $d^3k' \to d^3q$ and reversion of the common sign of the expression [1,2]. It is clear that the total probability of pair production by a photon given in Ref. [1] contains the same mistake as the probability of photon emission. The correct result has the same structure as the probability $W^{\pm}$ given by Eq. (21).

Our interpretation of electron-plane-wave interaction in terms of wave photons is based primarily on correspondence between exact theory and perturbation theory. Though the strong plane wave field is treated in our approach semiclassically and strictly speaking one cannot talk about photons, this concept is nevertheless very helpful and not contradictory. In agreement with it the electron can absorb 4-momentum from the wave by only discrete portions which are multiples of wave 4-vectors of one or another mode, and kinematic equations for both photon emission and pair production processes take such form as if the electrons involved in the process interact with real photons.

Our interpretation is quite traditional and does not differ from that used for the processes in a monochromatic wave. All partial probabilities can be classified in terms of wave photons without introducing ‘‘combination photons’’ proposed in Ref. [1] and for which there is no physical basis, as was explained in Sec. III.

Note that generally speaking only the total probability $W^{\pm}$ (21) can be represented as a sum of partial probabilities. For differential probability $dW^{\pm}/du\,d\varphi_0$ the same structure holds only when η is an irrational number. If η is rational it turns out that matrix elements corresponding to different numbers of photons n_i and n_i' obey the same kinematic relation if $n_1+\eta n_2=n_1'+\eta n_2'$. Therefore the differential probability involve, interference terms. These terms, however, vanish in the total probability. Indeed, the amplitudes (11) corresponding to the same kinematic equation but different numbers of absorbed photons have different phase factors and hence disappear after integration over φ_0.

Note also that when η is a rational number the differential probability due to the absence of the condition $n_2=n_2'$ exibits dependence on the phase shift φ (5) between the two modes of the external field. It is just this effect that forms the basis for the dependence of spatial pattern of electron radiation on the phase difference between the two modes of a two-frequency plane electromagnetic wave first discovered by Puntajer and Leubner [7].

ACKNOWLEDGMENTS

One of us (N.B.N.) thanks Professor K. T. McDonald, who drew his attention to Ref. [1]. This work was supported by Russian Foundation for Basic Research under Grant No. 97-02-16973.

[1] An Yu and H. Takahashi, Phys. Rev. E **57**, 2276 (1998).
[2] A. I. Nikishov and V. I. Ritus, Zh. Éksp. Teor. Fiz. **46**, 776 (1964) [Sov. Phys. JETP **19**, 529 (1964)].
[3] N. B. Narozhny, A. I. Nikishov, and V. I. Ritus, Zh. Éksp. Teor. Fiz. **47**, 930 (1964) [Sov. Phys. JETP **20**, 622 (1965)].
[4] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[5] A. I. Nikishov and V. I. Ritus, Zh. Éksp. Teor. Fiz. **47**, 1130 (1964) [Sov. Phys. JETP **20**, 757 (1965)].
[6] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed. (Pergamon Press, New York, 1982).
[7] A. K. Puntajer and C. Leubner, Opt. Commun. **73**, 153 (1989).